

The Ultimate Guide to Maths Mastery

The lowdown on Maths Mastery, why it's effective, and how it can transform Maths teaching and learning across your school



Who is this guide aimed at? ____

School leaders and Maths specialists who want to implement mastery teaching techniques in daily Maths lessons, but don't feel ready or may not want to fully invest in a scheme.

What will this guide do? _____

This guide will explain the benefits of mastery techniques used successfully in Asian classrooms. Many of these have been adapted to work well in British classrooms.

What is in this guide? _____

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Why mastery?

If your school is considering mastery teaching in Maths, but is unsure where to start, don't worry - yours is not the only one.

Until recently, teaching children to 'follow a process' had been at the core of Maths education. Explaining how or why something happened became merely an afterthought.

As long as a child got the answer correct, their comprehension of the Maths was less important. The problem was that, by not teaching pupils conceptually, we left them incapable of making the necessarily links required to problem solve (an increasing priority in the new Maths curriculum).

More importantly, it denied children the opportunity to truly enjoy and engage with Maths – as it became merely a memory test for many of them.

With the 2014 new curriculum, this began to change. Increasingly pupils are required to reason, problem solve, and discuss methods. This is where teaching for mastery shines, which is partly why so many schools have begun to make the switch.

If you're unsure about beginning a mastery path, or not yet persuaded by the approach, this resource will guide you through the key aspects of teaching for mastery in Mathematics. Starting with its' three fundamental aspects: conceptual understanding and procedural implementation, investing in your staff, and deeper understanding.

If you're one of the lucky schools to have received the government funded training and support, you will, we're sure, have considering implementing some form of mastery techniques in your school. If you're unsure where to start, this guide is perfect for you. We'll take you through mastery from the ground up, from setting and planning, to using bar models and journals in your classroom.

Finally, if your a school that has already implemented the approach, this guide is the perfect health check for your mastery pedagogy!

Conceptual understanding & procedural implementation

Using mastery techniques helps to break the cycle of rote learning. Furthermore, it provides children with the opportunity to grasp 'real' Maths, empowering them with problem solving skills and – perhaps more importantly – a sense of achievement.

"Using mastery techniques helps to break the cycle of rote learning. Furthermore, it provides children with the opportunity to grasp 'real' Maths." At the core of mastery is the teaching of concepts alongside their procedural algorithms. By investigating a variety of methods, children learn to demonstrate number sense by selecting the most efficient methods for the task. Arming children with strategies to tackle problems logically, using what they already know, is another vital of part of this style of learning.

Mastery is an investment in your staff_

Investing in mastery is an investment in your staff, the training will improve their teaching practice and, therefore, the teaching practice in your school. Here are the three main things you should keep in mind.

1. Expertise. There is a degree of pedagogical expertise required to facilitate this type of learning effectively, so investing in staff CPD is a brilliant way to enhance staff subject knowledge.

2. Confidence. Having the confidence to teach mastery is vital. Setting up a maths working party of enthusiastic staff members is one way to help champion the necessary shift towards teaching mastery.

3. Research. Organisations such as the NCETM (National Centre for Excellence in the Teaching of Mathematics) and MA (Mathematics Association) offer a range of courses. Send a few passionate teachers on a course with the view that they can disseminate what they learn to the rest of the school on return.

Small steps lead to deeper understanding _

Mastery is not a 'quick fix', and the whole school needs to be on board and committed. It will take time for teachers to learn how to effectively teach each topic. Additionally, pupil progress in lessons may be less dramatic when compared to the traditional teaching of Maths.

This can be seen in a positive light for three reasons:

- 1. The small steps in learning do not overwhelm pupils.
- 2. Every child can access the learning and the concepts are fully embedded before moving on.
- 3. Teachers can make formative assessments during lessons and through detailed marking, allowing the class to advance at the same time (see Grouping/ Setting).

These small increments of learning are fundamental to learning conceptually. If a 'jump' to the next concept is too large, children become lost and gaps start to form in their knowledge. This can be particularly evident in those with poor attendance. However, teaching mastery does account for this. These children are able to catch up as ample time is given to embed concepts.

Why mastery - summary Conceptual understanding is taught alongside procedural understanding. Teacher pedagogical knowledge needs to be supported. Small, ordered steps embed and build on learning. Ensure that the whole school are on board and 'champions' are in place to drive the new teaching style.

Grouping/setting

The debate about 'setting' children into ability groups is an ongoing one. Many feel that doing so is beneficial: it's undoubtedly easier to plan for and teach to a smaller spectrum of ability, and it does work well when teaching children to apply a set of rules.

However, mastery teaching is different. Here, the teacher no longer tells the pupils what to do; instead they facilitate learning by guiding the pupils to think for themselves. Communication is a vital part of this process. It is through peer discussion, and the proving and disproving of ideas, that metacognition happens (being aware of one's thought processes).

As such, peer discussion in mastery teaching and learning is key, which is why you should consider the advantages of the mixed ability classroom:

- There will be more fruitful discussions for lower ability children to participate in.
- The higher ability children have the opportunity to explain their methods to lower achievers which ensures they have ascertained a depth of understanding.

For more on the debate on setting in classrooms, read 'Mixed ability vs ability grouping: Where do you stand?'

How to group _

If a school is introducing a mastery initiative for the first time, then mixed ability groups can work well up to year 4. However, by year 5 and 6 the range of ability may be too wide to implement a totally mixed group at this stage. In this case, it might be possible to separate the highest or lowest achievers from the rest, however this will depend on the cohort.

Planning interventions to plug gaps in core knowledge is an effective way to ensure that the whole class can progress together. These could be based on an audit of each child's needs as opposed to one regular intervention group.

Having a 'Maths meeting' is an effective teaching model adapted from Shanghai schools. Here, the teacher is released for 30 minutes in the afternoon to support those pupils who didn't understand the concepts taught that day.

"Peer discussion in mastery teaching and learning is key, which is why you should consider the advantages of the mixed ability classroom."

Challenge rapid learners

Mastery learning happens in small steps which enables all children to move through key points of understanding together. Advanced learners are not left without challenge, as this can be achieved through higher–level questioning and extension tasks. Ideas on how to extend and support children are given in the 'Planning' and 'Questioning' sections.

Grouping/setting summary

- Mixed ability groupings to ensure good discussions and 'bouncing' of ideas. Seat pupils carefully so that partners can learn from each other.
- Hold daily 'Maths meetings' so that teachers have a chance to plug gaps for learners in their class.
- Challenge your advanced learners with tasks that develop depth and mastery. Whole class work on the same mathematics learning together.

How to plan a mastery lesson

Historically, government schemes (Numeracy Strategy, Numeracy Hour) limited the amount of flexibility teachers had in terms of when they taught each topic.

The new curriculum instead permits a freer approach, as long as a certain amount is covered by the end of each Key Stage. As such, teachers may spend a long time planning and creating resources with a wide range of differentiation.

However, with mastery it is possible to be flexible without the need for excessive planning.

So what should you teach, and when?

Having more autonomy to decide the direction of your lesson is a good thing, but it can leave teachers with questions such as:

- Which topic should be planned first?
- Is there a hierarchy of topics?
- How long should be spent on a topic?
- What are the non-negotiables for each lesson?
- Is there a sensible order of topics to maximise learning?

In answer to these questions there is no definitive order, but there are some concepts that must be taught prior to teaching others. Online tools can aid planning by mapping out lesson objectives. Maths No Problem is based on Singapore Maths and designed by experts. This is a subscription-based website, however it provides a breakdown of the small steps within each topic.

As the whole class works together, tasks should be presented in a format that all students can relate to, for example pictures or visual will support EAL and SEN children. Including pictures or visuals will support EAL and SEN children. The problems should be 'low threshold, high ceiling' (LTHC) activities that enable all pupils to access the learning, but have depth to cater for more advanced learners.

An example LTHC problem

I have 31 balloons, 5 of them blew away. How many balloons are left?



This Year 2 question looks at subtraction. For many children, 31 – 5 can be meaningless and confusing.

Renaming (or exchanging) the numbers in a context that is familiar to children gives the Maths meaning. Exploring the problem through discussion, and coming up with the number

sentence as a class, means children are more likely to access the Maths.

Allowing children to do this independent of the teacher also has merit. The teacher can circulate the room, assessing learning and asking questions to support learners or challenge their thinking.

Differentiation and assessment

For more **advanced learners**, challenge can be achieved by proving their answers or explaining a method that they have used, or thinking of an alternative method to solve the above problem.

For **struggling learners**, they may approach the above problem by counting the number of remaining balloons or using manipulatives like base ten (see 'Manipulatives')

Children need ample time to explore and demonstrate 'exchange' with physical equipment. Once this has happened, learning can be linked to the column method. At each stage, pupils should be able to explain what is happening by referring back to the balloon "Problems should be 'low threshold, high ceiling' activities that enable all pupils to access the learning, but have depth to cater for more advanced learners."

context. This is only possible if children have a solid understanding of place value and the base ten system - without this, the 'crossing out' and 'carrying over' just becomes a learnt procedure.

Differentiation in mastery sessions is not presented on different worksheets, but through teacher expectation and challenge. The positives of this are twofold. Less time-consuming planning and boosted confidence for lower ability children (as the work isn't tiered).

For quick finishers, in depth questions such as these can be used to further their learning:

- "Can you come up with a 'brand new' technique to solve the problem?"
- "Can you write a leaflet to explain what you were working on today for an absent classmate?"

By assessing what they have written, teachers gain insight into pupils' depth of understanding and highlight any possible misconceptions. Activities from the <u>NCETM</u> <u>National Curriculum Assessment Materials</u> are useful for assessing children at mastery.

Knowing the lesson inside out

To teach for mastery means being prepared, and teachers need to take into account what the non-negotiables are for that lesson:

• Subject knowledge. How best to teach the concepts with sound pedagogy.

- Awareness. Knowing the common misconceptions in the topic.
- Technical terms. Introducing the correct terminology
- **Expectations**. Having an expectation that children answer in whole sentences, using mathematical language
- **Feedback**. If possible, encourage staff to plan together and give feedback of practices that were or were not successful.

Spend ample time on concepts _

'Over-teach' concepts, especially the core areas of:

- Number and place value
- The four operations
- Times table and division facts

Seeing patterns and structures in Maths is the key to being able to generalise thinking and enables children to make sense of topics such as fractions and decimals. Children will be unable to see, for example, the relationship between equivalent fractions if they don't know their times tables.

Planning - summary_

- Map out learning objectives so that learning is progressive in small steps.
- All children work on the same task and are supported and/or challenged.
- **C** Design tasks that are meaningful and which children can relate to.
- Know the lesson well and be aware of potential misconceptions.
- Assess mastery and depth rather than coverage.
- **Over-teach' key areas of learning to help embed and link knowledge together.**
- **Use correct terminology and encourage pupils to answer questions in full sentences.**

"With mastery it is possible to be flexible without the need for excessive lesson planning."

Questioning

A hugely important part of a mastery classroom is questioning. More traditional 'chalk and talk' lessons can be transformed into an excited discussion by incorporating good questions.

In successful mastery classrooms, it is common to see hands thrown in the air in order to prove or disprove what has been asked. A cleverly asked question will see children 'struggling' between their own preconceived ideas and the logic that may be in front of them.

The peer discussions that take place as a result of questions posed are invaluable as a way for teachers to assess levels of understanding, and as a platform for pupils to learn from one another

True mastery is demonstrated when two elements are combined:

- The reasoning and justifying of a concept.
- The correct use of technical language needed to convey a message.

What does this look like in the classroom?

- **Noise level**: This will be good noise pupils on task trying to prove their point during peer or group discussions.
- **Teacher circulating:** Asking appropriate questions will facilitate learning and deeper thinking.
- **Unbiased attitude**: It is important that the teacher neither validates nor rejects an answer or a theory for two reasons:

If the answer is confirmed correct, the rest of the class no longer need to think for themselves. An alternative approach is to ask several children what they think the answer is before asking, 'How do you know?'

It is up to the whole class to justify and reason the theory correct or incorrect. Teachers avoid 'telling' the students the answers, and the pupils have to work out why it must be correct – a more powerful type of learning that helps link concepts together.

"A cleverly asked question will see children 'struggling' between their own preconceived ideas and the logic that may be in front of them."

What questions to ask?

Some generic questions to ask that are perfect to help assess learning include:

- How do you know?
- Can you prove it?
- Can you come up with a different method?
- What do you notice?
- Will it always do that, and why?
- What happens if?
- Does your answer seem reasonable? Why/why not?

Thought must be given to the types of questions teachers ask. These need to be tailored to support or challenge learners.

Teachers require in-depth knowledge of the topic - something that may not come naturally but can be learnt. To aid this, it is good practice for teachers to share things that went well and things that could be improved. Learning from each other and not being afraid of making mistakes are significant parts of mastery (see Growth Mindset section).

'Teacher Research Groups' are very common in Asian countries - however, they are given dedicated time for these meetings to happen. Nonetheless, it is possible to promote such feedback informally.

One of the most important parts of challenging higher ability children is questioning. The power of a simple question should not be underestimated. For example, ask questions such as:

- 'What do you notice?'
- 'Will it always happen?'

These help to promote depth of understanding. Additionally, topic specific questions are necessary.

Lack of knowledge shouldn't deter a teacher from asking questions beyond their own understanding. In fact, allowing advanced learners to discover and explore, can lead to the teacher learning something. This is beneficial as children enjoy seeing teachers who don't know all the answers.

Questioning - summary_

- Purposeful discussions are vital for creating an enthusiastic mastery classroom.
- Peer discussions are integral to assessment and learning.
- More effective learning takes place when the teacher doesn't immediately accept or reject an answer.
- Higher level questioning can be used to challenge advanced learners. Teachers should be brave and not afraid to learn with the class.

Growth mindset

The power of having an open mind is a teaching and learning asset. A growth mindset from a learner's point of view is the self belief that you can improve at anything if you work at it, and that the brain gets 'stronger' as you do.

Pupils with a growth mindset:

- Thrive on challenge •
- Are resilient to setbacks
- Show a great deal of perseverance

A fixed mindset is the opposite:

- Those that give up easily
- Say things like, "I can't do it!" •
- Won't even attempt something new
- Are afraid of making mistakes
- Never answers in class unless they are 100% sure •
- Rub out mistakes in their books

Research shows that pupils with a fixed mindset are less likely to reach their full academic potential.

How can teaching for mastery help? _

A mastery class adopts a 'can do' attitude. There is an underlying message that all children can learn Maths and that no one is born being 'good' or 'bad' at Maths.

Mastery aims to promote a growth mindset, providing a safe environment where mistakes are celebrated as a key to unlocking new learning. Through class discussions, misconceptions are challenged by concrete proof. This provides evidence that the 'understanding' a child had before must have been incorrect.

It is vital to approach this in a supportive way, so no child is made to feel uncomfortable or stupid.

As mastery is about conceptual learning, children begin to make links across Maths. As their understanding improves, self-belief and confidence grow. Therefore, over time children begin to look forward to the challenge of Maths as opposed to be daunted by the prospect.

Classdojo have some great videos to use to help encourage a more positive mindset these work well during PSHE.

The story of the butterfly is a good metaphor for mastery: If the butterfly does not struggle as it emerges from its chrysalis, it will never fly. This demonstrates how struggling with learning helps us to be better. Encouraging a positive attitude towards learning helps



Growth mindset - summary

- Fixed mindsets lead to a fear of failure and a need to prove intelligence at all opportunities.
- **Teaching for mastery promotes a growth mindset beneficial throughout school.**
- **C** Encourage growth mindset using resource ideas.

Manipulatives

In mastery, it is widely accepted that children learn best through a Concrete Pictorial Abstract approach (CPA). The initial concrete phase brings concepts to life by using physical objects. Manipulatives play a key role in this as they are the concrete resources used to support pupils thinking as they explore abstract ideas. Using something 'real' to make sense of the Maths takes away the need to imagine or visualise at the early stage of learning a new concept.

Read this blog post on how hands on manipulatives transform primary Maths for additional information and a list of the top ten manipulatives schools can invest in.

How to ensure you don't abuse use of manipulatives .

- Investing, not only in resources, but also on training will ensure that manipulatives are used effectively.
- It is important that manipulatives are not solely relied upon there is a danger that over-reliance on equipment will hinder progress.
- With experience, teachers will gauge when is most appropriate to use materials, which materials to use and when they should be removed. This can also be personalised to individual children depending on their needs.

Choosing equipment _____

Overall, you should allow autonomy when choosing equipment. When children are familiar with the manipulatives and how to use them, give them a chance to select their own choice of equipment to represent the problem they are to solve. This is helpful because:

- The teacher can see how well the student has understood the concept by assessing the relevance of the resources chosen.
- It promotes high-level thinking pupils have to ensure that the resources chosen are made relevant to the problem and represent it adequately.

Manipulatives - summary _____

- Manipulatives provide a positive impact when teaching for mastery
- Investing in equipment, but also staff training is vital
- Con't over use manipulatives you don't want children become dependent on them
- Allow pupils to self-select equipment at times to assess learning

Anchor charts

Anchor charts are posters based on key concepts in a lesson and help children to 'anchor' their learning to these displays. They can be created during the lesson as learning unfolds or when children share their methods and ideas. Anchor charts can also be created post-lesson and used as a learning aid in subsequent lessons.

They should be constructed with the pupils and can be made personal by adding the name of the author: e.g. 'David's method'. They then serve as a memoire to learning and should be displayed prominently in the classroom and referred back to when appropriate.

Anchor charts are easily created by sticking a piece of sugar paper onto the whiteboard. The class teacher and a well-prepped TA can write and draw out key ideas. Children can help by writing their own methods and explanations.

What should anchor charts look like?

There are no rules to dictate what an anchor chart must look like, however:

- It should be clear and precise.
- It should be colourful and neat.
- Use of diagrams and pictures ensures that it is easier to read and understand.
- Include a challenge or an in-depth question can promote thinking in learners.
- Avoid making the posters text heavy as pupils will not want to read it.

Here are some examples of what an anchor chart might look like:



When to make an anchor chart?

Anchor charts should be utilised whenever there is a purpose for having one displayed in the classroom. This may differ year on year as it is somewhat dependent on the cohort of children.

If there is limited space on the walls, a 'washing line' display can be used. Alternatively, arrange the charts so that only the headings can be seen and the relevant pages can been 'flipped' to when necessary.

Plan for it ____

A lesson plan about the anchor chart is not required, but thinking about what should be included is useful. It may not always turn out exactly as planned, but it will help knowing how pictorial representations could be used and what challenge could be incorporated.

Anchor charts - summary _____

Anchor charts are:

- Created with students to log key ideas and learning
- G Used as a reference tool
- Created by teachers, children and TAs
- Thought about beforehand so that they showcase all the necessary information

Journalling

A huge part of Curriculum 2014 required students to demonstrate reasoning and fluency in Maths, but most teachers had difficulty evidencing this – particular if work that was set was just a page of calculations. A way to combat this was to introduce the 'Maths Journal' – a book dedicated to the thoughts and justifications of the individual pupil.

Here, children can log and articulate the different methods and ideas for solving a problem, enabling teachers to assess what is going on inside the child's head. Journalling could be considered a 'window into the child's thinking' and where misconceptions arise, they can be dealt with in a meaningful way.

5 things your pupils might journal

- **1. At the start of a 'new' topic** children can journal what they already know and what they would like to find out.
- **2. As the topic progresses**, children may journal about their thought processes and practices that they have used to come to a solution.
- **3.** They may also journal about the different methods, techniques and strategies that they have come across, explaining their understanding.
- **4. When looking at various methods**, children may want to comment on the most efficient and explain why the efficacy is dependent on the numbers used.
- 5. Towards the end of the topic, children could log what they have learnt in the unit of work and use examples to demonstrate their learning.

What they log is entirely up to the teacher, however their journals should provide teachers with a clear path of learning and progress that has taken place for each individual child.

When, why and how to journal?

Daily journaling – All students should keep a journal, but when to write in it depends on what learning the teacher wants to log. Daily journalling is recommended if different concepts are being introduced each day.

Assess understanding – Aside from being an assessment vehicle, journalising also encourages metacognition as the pupil is being forced to think again about the learning and how best to record it.

Have high standards – One of the fundamentals of journalling is that ideas and explanations are very clear. Precise language and diagrams should be used too. With advance learners, the expectations and the quality of work produced may be higher than that of a struggling learner, however ideally teachers shouldn't accept mediocrity – even lower ability children are able to use representations to explain what they are thinking.

Help with vocabulary – Producing a list of keywords with the class will help children focus their work further.

Use of pictures and diagrams

Pictorial representations are just as vital as concrete materials. These are particularly helpful for engaging EAL pupils, SEN pupils and visual learners, as well as being a platform to express their understanding. Initially, the teacher may need to model different visual representations but as the children get used to using them, they may well go on to create their own. There is no right or wrong model as long as it expresses accurately an explanation.

Aside from the ongoing assessment that takes place during a lesson, journals are a fantastic source of formative assessment. However, this can only work effectively if books are marked and kept up-to-date.

A typical journalling session should only last 10–15 minutes, so there shouldn't be copious amounts of marking. However, teacher feedback should help the pupil to identify an underlying misconception, whilst next step marking can promote depth of understanding.

Here are some examples of Maths journals:



Journalling summary

- Journals evidence children's reasoning and fluency and the progress made in a unit of work.
- Encourage pictures and diagrams as well as correct terminology and clear explanations.
- **Marked daily to inform planning and identify individual needs.**
- **C** Teacher decides on what and when to journal.

Bar modelling

Bar models were developed and used in Singapore and are an effective visual tool to help students break down, understand and solve a problem. Children need plenty of exposure to the different types of bar models before they can use them productively.

Through mastery learning in Maths, children should first learn simple bar models to show addition and subtraction, then multiplication and division. Gradually they build more elaborate models to solve more complex problems.

What are bar models?

Bar models are rectangular bars drawn to represent a scenario posed in a worded problem. These rectangles provide a way to 'see' the steps and the appropriate operations needed to solve the problem. They model the worded problem in its most simplistic state, removing all words except for the key information. This allows the student to deal with one section of the worded problem at a time, and gives space to decide whether the piece of information is important in the context.

For EAL and SEN learners, this visual image is powerful, as they no longer need to worry about long sentences which they may find difficult to comprehend – they can now see what they have to do.

An example of bar modelling can be seen to the right:



Bar modelling - summary

- Bar models give children an entry point into a worded problem.
- They help to break down the problem into more manageable chunks.
- A tool to help students visualise the problem and work out the steps to solve and the operation(s) to use.
- C Allows EAL and SEN children to 'see' what is being asked of them.

Further reading

Books _

- Bar Modelling: A Problem-solving Tool (Dr Yeap Ban Har)
- Mathematics Counts (The Cockcroft Report)
- Mathematics Explained for Primary Teachers (Derek Haylock)
- Teaching of Whole Numbers (Dr Yeap Ban Har & Douglas Edge)
- Teaching of Fractions (Douglas Edge & Dr Yeap Ban Har)



Yeap Ban Har of Maths No Problem

Teaching of Decimals (Douglas Edge & Dr Yeap Ban Har)



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