Released January 2018

Year 5

Small Steps Guidance and Examples

Block 2 - Fractions

White Rose Maths

Year 5 – Yearly Overview

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number – Place Value			Number – Addition and Subtraction		Statistics		Number – Multiplication and Division		Perimeter and Area		Consolidation
Spring	Number – Multiplication and Division			Number – Fractions						Number – Decimals & Percentages		Consolidation
Summer	Number – Decimals				Geometry- Properties of Shapes			Geometry- Position and Direction	Measurement- Converting Units		Measures Volume	Consolidation

Week 4 to 9 – Number: Fractions

Overview

Small Steps

- Equivalent fractions
- Improper fractions to mixed numbers
- Mixed numbers to improper fractions
- Number sequences
- Compare and order fractions less than 1
- Compare and order fractions greater than 1
- Add and subtract fractions
- Add fractions within 1
- Add 3 or more fractions
- Add fractions
- Add mixed numbers
- Subtract fractions
- Subtract mixed numbers
- Subtract – breaking the whole

NC Objectives

Compare and order fractions whose denominators are multiples of the same number.

Identify, name and write equivalent fractions of a given fraction, represented visually including tenths and hundredths.

Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements >1 as a mixed number [for example $\frac{2}{5}+\frac{4}{5}=\frac{6}{5}=1\frac{1}{5}$]

Add and subtract fractions with the same denominator and denominators that are multiples of the same number.

Week 4 to 9 – Number: Fractions

Overview

Small Steps

- Subtract 2 mixed numbers
- Multiply unit fractions by an integer
- Multiply non-unit fractions by an integer
- Multiply mixed numbers by integers
- Fraction of an amount
- Using fractions as operators

NC Objectives

Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

Read and write decimal numbers as fractions [for example $0.71 = \frac{71}{100}$]

Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Equivalent Fractions

Notes and Guidance

Children recap on learning from year 4. They explore equivalent fractions using models and concrete representations.

They use models to make the link to multiplication and division. Children will then be able to apply the abstract method to find equivalent fractions.
It is important children have the conceptual understanding before moving in to just using an abstract method.

Mathematical Talk

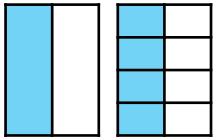
What equivalent fractions can we find by folding the paper? How can we record these?

What is the same and what is different about the numerators and denominators in the equivalent fractions?

How does multiplication and division help us find equivalent fractions? Where can we see this in our model?

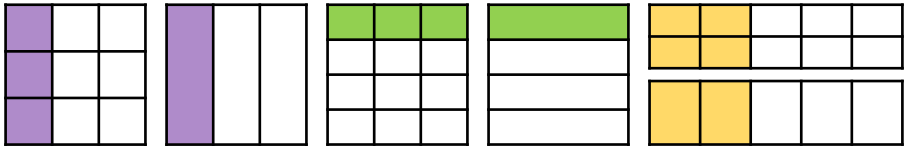
Varied Fluency

1 Take two pieces of paper the same size.
Fold on piece into two equal pieces.
Fold the other into eight equal pieces.
What equivalent fractions can you find?

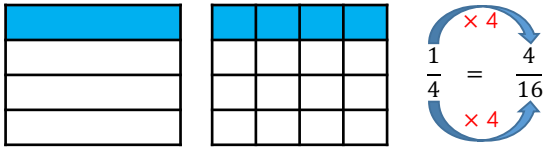


$\frac{1}{2} = \frac{4}{8}$

Use the models to write equivalent fractions.



2 Emma uses the models and her multiplication and division skills to find equivalent fractions.



Use this method to find equivalent fractions to $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$ where the denominator is 16

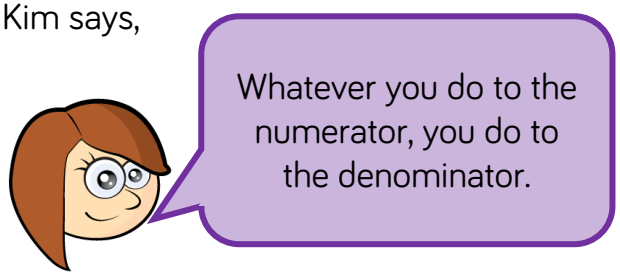
3 Emma uses the same approach to find equivalent fractions for these fractions. How will her method change?

$\frac{4}{12} = \frac{\square}{3}$ $\quad$ $\frac{6}{12} = \frac{\square}{4}$ $\quad$ $\frac{6}{12} = \frac{\square}{2}$

Equivalent Fractions

Reasoning and Problem Solving

Kim says,



Here are the equivalent fractions she has found for $\frac{4}{8}$:

$\frac{4}{8} = \frac{8}{16}$ $\quad \frac{4}{8} = \frac{6}{10}$

$\frac{4}{8} = \frac{2}{4}$ $\quad \frac{4}{8} = \frac{1}{5}$

Does Kim's method work? Explain why.

Kim's method doesn't always work. It works when multiplying or dividing both the numerator or denominator but not when adding or subtracting the same thing to both.

Martin thinks you can only simplify even numbered fractions because you keep on halving the numerator and denominator until you get an odd number.

Do you agree?
Explain your answer.

Martin is wrong.

For example $\frac{3}{9}$ can be simplified to $\frac{1}{3}$ and these are all odd numbers.

Here are some fraction cards. All of the fractions are equivalent.

$\frac{4}{A}$ $\quad \frac{B}{C}$ $\quad \frac{20}{50}$

A + B = 16
Calculate the value of C.

A = 10
B = 6
C = 15

Improper to Mixed Numbers

Notes and Guidance

In this step, children convert from improper fractions to mixed numbers for the first time. An improper fraction is a fraction where the numerator is larger than the denominator. A mixed number is a whole number alongside a fraction.

It is important for children to see this process represented visually to allow them to make the connections between the concept and what happens in the abstract.

Mathematical Talk

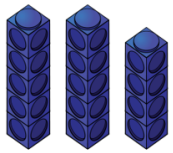
Why are we grouping the cubes into 5s?

How many fifths are there in a whole?

What do you notice about the improper fraction and the mixed number?

Varied Fluency

1 Claire converts the improper fraction $\frac{14}{5}$ into a mixed number using cubes. She groups the cubes into 5s, then has 4 left over.

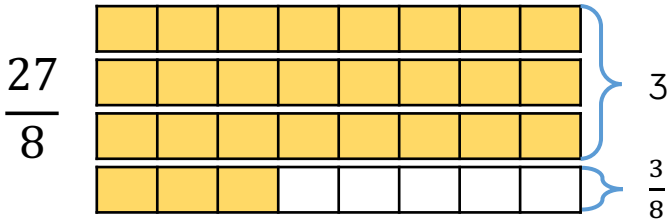


$\frac{5}{5}$ is the same as ☐ $\frac{10}{5}$ is the same as ☐

$\frac{14}{5}$ as a mixed number is ☐ ☐/☐

Use Claire's method to convert $\frac{19}{3}$, $\frac{19}{4}$, $\frac{19}{5}$ and $\frac{19}{6}$

2 Steve converts the improper fraction $\frac{27}{8}$ into a mixed number using bar models.

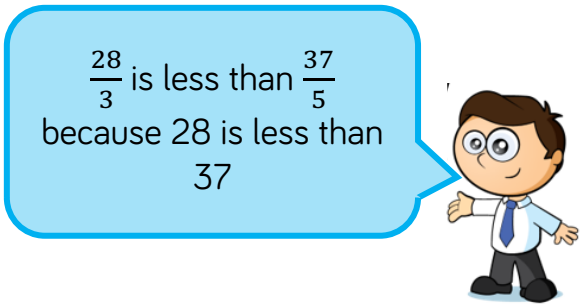


Use Steve's method to convert $\frac{38}{8}$, $\frac{27}{6}$, $\frac{47}{7}$ and $\frac{32}{4}$

Improper to Mixed Numbers

Reasoning and Problem Solving

William says,



Do you agree?
Explain why.

William is incorrect because $\frac{28}{3}$ equals $9\frac{1}{3}$ and $\frac{37}{5}$ equals $7\frac{2}{5}$

Spot the mistake

- $\frac{27}{5} = 5\frac{1}{5}$
- $\frac{27}{3} = 8$
- $\frac{27}{4} = 5\frac{7}{4}$
- $\frac{27}{10} = 20\frac{7}{10}$

Can you find the correct answers?

Answers should be:

- $5\frac{2}{5}$ (incorrect number of fifths)
- 9 (incorrect whole)
- $6\frac{3}{4}$ (still have an improper fraction)
- $2\frac{7}{10}$ (incorrect number of wholes)

Mixed Numbers to Improper

Notes and Guidance

Children now convert from mixed numbers to improper fractions using concrete and pictorial methods to understand the abstract method.

Ensure children always write their working alongside the concrete and pictorial representations so they can see the clear links.

Mathematical Talk

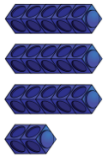
How many quarters/halves/eighths are there in a whole?

What do you notice about the whole number and the denominator?

What happens to the whole number and the numerator? Why?

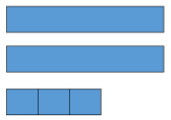
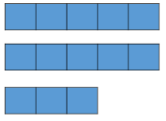
Varied Fluency

1 Beth uses cubes to help her convert from mixed numbers to improper fractions.

Step 1: Start with mixed number	Step 2: Build the mixed number using cubes. Think carefully about how many parts make a whole.	Step 3: Count the number of cubes to find the numerator of the improper fraction
$3\frac{2}{5}$		$3\frac{2}{5} = \frac{17}{5}$

Use Beth's method to convert $2\frac{2}{3}$, $2\frac{2}{4}$, $2\frac{2}{5}$ and $2\frac{2}{6}$

2 Sam uses bar models to convert a mixed number into an improper fraction.

Step 1: Draw representation of mixed number.	Step 2: Split representation into equal parts.	Step 3: Count the number of equal parts to find the numerator of the improper fraction
		$2\frac{3}{5} = \frac{13}{5}$

Use Sam's method to convert $2\frac{1}{6}$, $3\frac{2}{3}$, $1\frac{4}{5}$ and $7\frac{5}{12}$

Mixed Numbers to Improper

Reasoning and Problem Solving

Three children have converted $3\frac{2}{5}$ into an improper fraction.

Child A

$$3\frac{2}{5} = \frac{32}{5}$$

Child B

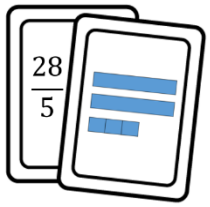
$$3\frac{2}{5} = \frac{17}{15}$$

Child C

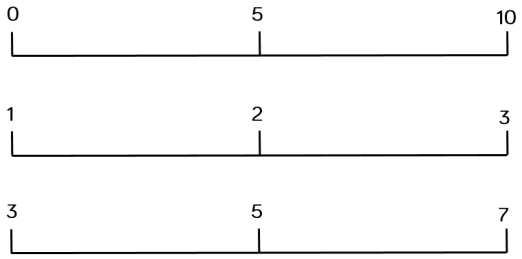
$$3\frac{2}{5} = \frac{7}{5}$$

Child A has just put the 3 on the numerator
Child B has multiplied both the numerator and the denominator by 3
Child C has only used 1 whole not all 3 wholes.

You will need: Mixed number and improper fraction representation cards.



Work with your partner and take it in turns to take a card and reason about where your card may go.
Avoid converting straight away and try to reason first.



Teacher notes:

Create mixed and improper fraction cards based on any areas of misconceptions observed during any previous learning.

Number Sequences

Notes and Guidance

Children will count up and down in a given fraction. They will continue to use visual representations to help them explore number sequences.

They will also find missing fractions in a sequence and determine whether the sequence is increasing or decreasing and by how much.

Mathematical Talk

What other start numbers could we begin with?
Will your sequence increase or decrease?
How much will it go up or down by each time?

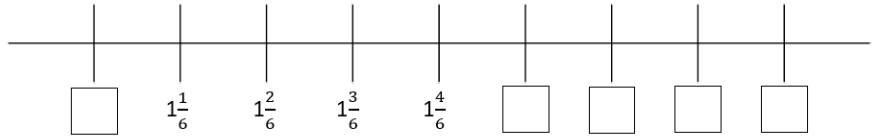
If my sequence is decreasing by ___ and the ___ number is worth ____, what numbers will be in my sequence?

Varied Fluency

1 Use the counting stick to count up and down in these fractions.

- Start at 0 and count up in steps of $\frac{1}{4}$
- Start at 2 and count down in steps of $\frac{1}{3}$
- Start at 1 and count up in steps of $\frac{2}{3}$

2 Complete the missing values on the number line.



3 Find the missing fractions in the sequences.

$\frac{3}{4}, \square, 1\frac{3}{4}, 2\frac{1}{4}$

$\square, 3\frac{1}{3}, \square, 2\frac{2}{3}$

$\square, 5\frac{1}{2}, 5\frac{7}{10}, 5\frac{9}{10}$

$\frac{3}{5}, \square, \square, 3$

Number Sequences

Reasoning and Problem Solving

Three children are counting in fractions.

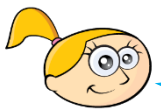
Chantelle

$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}$

Aidan

$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}$

Ellie



$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, 1\frac{1}{4}, 1\frac{2}{4}, 1\frac{3}{4}$

Who is counting correctly?

They are all correct, they are all counting in quarter. Aidan has simplified all answers and Ellie has converted improper fractions mixed numbers.

Jasmine and Dayle are counting in fractions.
Jasmine counts up in thirds and starts at 0
Dayle counts down in sixths and starts at 5

How many numbers do they each say before they say the same number?

Jasmine will says 10 numbers and Dayle will say 13 numbers and they will both have said 3

Compare & Order (Less than 1)

Notes and Guidance

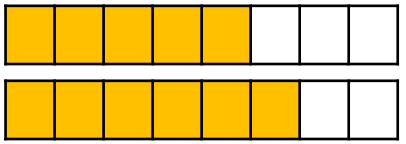
Children build on their equivalent fraction knowledge to compare and order fractions less than 1 where the denominators are multiples of the same number.
It is important that children are able to draw models so that they can directly compare them.
Children need to find the common denominator in this step. They may also investigate finding a common numerator.

Mathematical Talk

How does a bar model help us to visualise the fractions?
Should both of our bars be the same size? Why? What does this show us?
If the numerators are the same, how can we compare our fractions?
If the denominators are the same, how can we compare our fractions?
Do we always have to find a common denominator? Can we find a common numerator?

Varied Fluency

1 Use bar models to compare $\frac{5}{8}$ and $\frac{3}{4}$



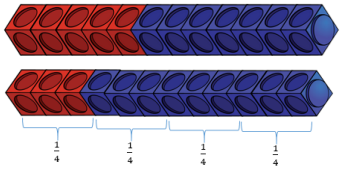
$\square > \square$

$\square < \square$

Use this method to help you compare:

$\frac{5}{6}$ and $\frac{2}{3}$ $\quad$ $\frac{2}{3}$ and $\frac{5}{9}$ $\quad$ $\frac{7}{16}$ and $\frac{3}{8}$

2 Use cubes to help you compare $\frac{1}{4}$ and $\frac{5}{12}$



$\square > \square$

$\square < \square$

Use this method to help you compare:

$\frac{6}{7}$ and $\frac{15}{21}$ $\quad$ $\frac{4}{9}$ and $\frac{11}{27}$ $\quad$ $\frac{9}{16}$ and $\frac{7}{8}$

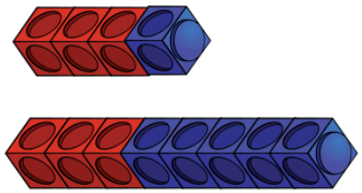
3 Order the fractions from greatest to smallest:

$\frac{3}{12}, \frac{3}{4}$ and $\frac{3}{16}$ $\quad$ $\frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12}$ $\quad$ $\frac{4}{7}, \frac{13}{14}$ and $\frac{19}{28}$

Compare & Order (Less than 1)

Reasoning and Problem Solving

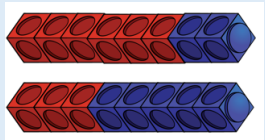
Ash makes $\frac{3}{4}$ and $\frac{3}{8}$ out of cubes.



He thinks that $\frac{3}{8}$ is equal to $\frac{3}{4}$

Do you agree?
Explain your answer.

Possible answer: I disagree with Ash because the two wholes are not equal. He needs to convert $\frac{3}{4}$ to $\frac{6}{8}$ in order to compare the fractions. If he does this he will see that $\frac{3}{4}$ is biggest.



Always, sometimes, never

If one denominator is a multiple of the other you can simplify the fraction with the larger denominator to make the denominators the same.

E.g. $\frac{1}{4}$ and $\frac{9}{12}$ can be simplified to $\frac{1}{4}$ and $\frac{3}{4}$

Prove it.

Sometimes

This can happen as long as the numerators are multiples of the same number too. It cannot work for some fractions e.g. $\frac{8}{15}$ and $\frac{3}{5}$

15 is a multiple of 5 but 8 is not so it does not work for this case.

Compare & Order (More than 1)

Notes and Guidance

Children use their knowledge of ordering fractions less than 1 to help them compare and order fractions greater than 1

They use their knowledge of common denominators to help them.

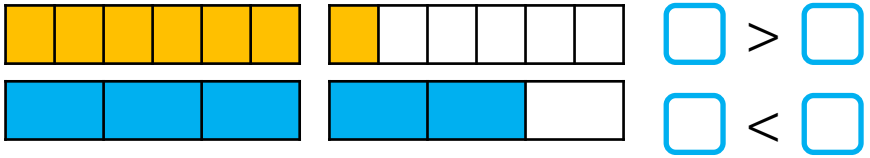
Children will compare both improper fractions and mixed numbers during this step.

Mathematical Talk

How can we represent the fractions?
How does the bar help us see which fraction is the greatest?
Can we use our knowledge of multiples to help us?
Can you predict which fractions will be greatest? Explain how you know.
When we are comparing mixed numbers what can we do to the bars to help us see each fractions?

Varied Fluency

1 Use bar models to compare $\frac{7}{6}$ and $\frac{5}{3}$



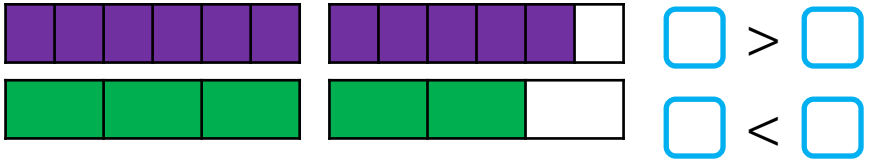
$\square > \square$

$\square < \square$

Use this method to help you compare:

$\frac{7}{2}$ and $\frac{9}{4}$ $\frac{11}{6}$ and $\frac{13}{3}$ $\frac{9}{4}$ and $\frac{17}{8}$

2 Use a bar model to compare $1\frac{2}{3}$ and $1\frac{5}{6}$



$\square > \square$

$\square < \square$

Use this method to help you compare:

$1\frac{3}{4}$ and $1\frac{3}{8}$ $1\frac{5}{8}$ and $1\frac{1}{2}$ $2\frac{4}{7}$ and $2\frac{9}{14}$

3 Order the fractions from greatest to smallest:

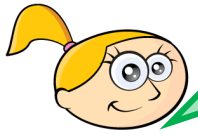
$\frac{8}{5}$, $\frac{11}{10}$ and $\frac{17}{20}$ $1\frac{2}{3}$, $1\frac{7}{24}$ and $1\frac{5}{12}$ $1\frac{3}{8}$, $1\frac{11}{16}$ and $\frac{19}{28}$

Compare & Order (More than 1)

Reasoning and Problem Solving

Lucy and Sinead both have two identical pizzas each.

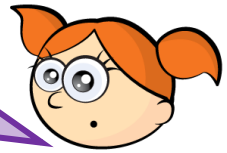
Lucy says,



I have cut each pizza into 6 equal pieces and eaten 8

Sinead says,

I have cut each pizza into 9 equal pieces and eaten 15



Who ate the most pizza?

Use a drawing to support your answer.

Sinead ate the most pizza because $\frac{15}{9}$ is greater than $\frac{8}{6}$

Lottie looks at the fractions $1\frac{7}{16}$ and $1\frac{3}{4}$

She says,

$1\frac{7}{16}$ is greater than $1\frac{3}{4}$ because the numerator is larger.

Do you agree?

Explain why using a model.

Possible answer:

I do not agree because $1\frac{3}{4}$ is equivalent to $1\frac{12}{16}$ and this is greater than $1\frac{7}{16}$

Add & Subtract Fractions

Notes and Guidance

Children recap their year 4 understanding and add and subtract fractions with the same denominator.

They use bar models to support understanding of adding and subtracting fractions.

Mathematical Talk

How many equal parts do I need to split my bar into?

Can you convert the improper fraction into a mixed number?

How can a bar model help you balance both sides of the equals sign?

Varied Fluency

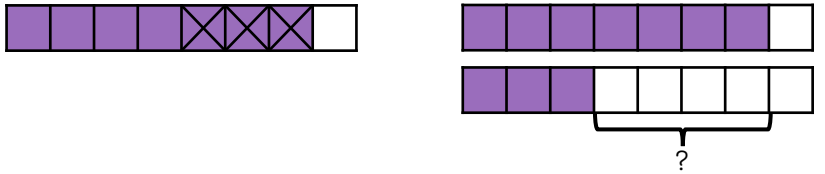
1 Here is a bar model to calculate $\frac{3}{5} + \frac{4}{5}$

$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$

Use a bar model to solve the calculations:

$\frac{3}{8} + \frac{3}{8}$ $\frac{5}{6} + \frac{1}{6}$ $\frac{5}{3} + \frac{5}{3}$

2 Here are two bar models to calculate $\frac{7}{8} - \frac{3}{8}$



What is the difference between the two methods?

Use your preferred method to calculate:

$\frac{5}{8} - \frac{1}{8}$ $\frac{9}{7} - \frac{4}{7}$ $\frac{5}{3} - \frac{5}{3}$

3 Calculate:

$\frac{3}{7} + \frac{5}{7} = \frac{\square}{\square} + \frac{4}{7}$ $\frac{9}{5} - \frac{5}{5} = \frac{6}{5} - \frac{\square}{\square}$ $\frac{2}{3} + \frac{\square}{\square} = \frac{11}{3} - \frac{4}{3}$

Add & Subtract Fractions

Reasoning and Problem Solving

How many different ways can you balance the equation?

$$\frac{5}{9} + \frac{\square}{9} = \frac{8}{9} + \frac{\square}{9}$$

Possible answers:

$$\frac{5}{9} + \frac{3}{9} = \frac{8}{9} + \frac{0}{9}$$

$$\frac{5}{9} + \frac{4}{9} = \frac{8}{9} + \frac{1}{9}$$

$$\frac{5}{9} + \frac{5}{9} = \frac{8}{9} + \frac{2}{9}$$

Any combination of fractions where the numerators add up to the same total on each side of the equals sign.

A chocolate bar has 12 equal pieces.

Sami eats $\frac{5}{12}$ more of the bar than Hafsah.

There is one twelfth of the bar remaining.

What fraction of the bar does Hafsah eat?

Sami eats $\frac{8}{12}$ of the chocolate bar and Hafsah eats $\frac{3}{12}$ of the chocolate bar.

Add Fractions within 1

Notes and Guidance

Children add fractions with different denominators for the first time. The denominators are multiples of one another.

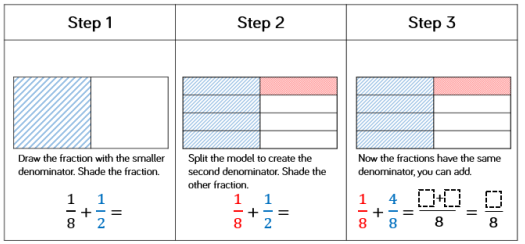
It is important that children see this represented visually so they can make connections with the abstract.

Mathematical Talk

How can we convert ___ into ___?
How can we convert thirds into fifteenths?
What do you think the common denominator might be? Why?
Could it be anything else?
What do you notice about the denominators?
Can you simplify your answer?

Varied Fluency

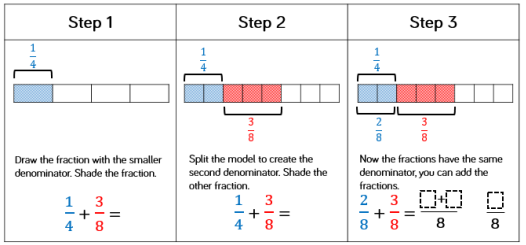
1



Use the area model to solve :

$\frac{1}{4}+\frac{3}{8}$ $\frac{2}{3}+\frac{1}{6}$ $\frac{7}{10}+\frac{1}{5}$

2



Use the bar model to solve:

$\frac{1}{6}+\frac{5}{12}$ $\frac{2}{9}+\frac{1}{3}$ $\frac{1}{3}+\frac{4}{15}$

3 Use your preferred method to solve:

$\frac{2}{3}+\frac{3}{15}$ $\frac{7}{12}+\frac{1}{4}$ $\frac{4}{7}+\frac{7}{21}$ $\frac{5}{14}+\frac{2}{7}$

Add Fractions within 1

Reasoning and Problem Solving

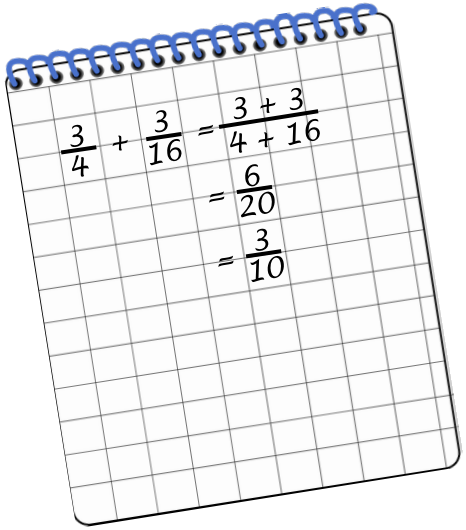
$$\frac{5}{16} + \frac{\square}{8} = \frac{15}{16}$$

5

$$\frac{\square}{20} + \frac{7}{10} = \frac{17}{20}$$

3

Mary-Kate solved this calculation:



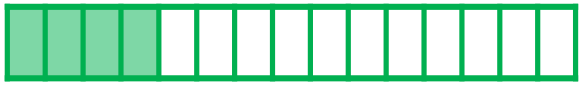
Can you spot and explain her mistake?

Mary-Kate is wrong because she has just added the numerators and the denominators. When adding fractions you need to find a common denominator.

Two children are solving $\frac{1}{3} + \frac{4}{15}$

Emma starts by drawing this model:

Kate starts by drawing this model:



Can you explain each person's method and how they would complete the question?
Which method do you prefer and why?

Emma has split her bar into thirds shaded one, she then needs to split each third into 5 equal pieces and shade a further 4
Amy has started with fifteenths and shaded in 4 pieces, she then needs to shade in another 5 pieces as $\frac{1}{3} = \frac{5}{15}$
They will both get an answer of $\frac{9}{15}$ which simplifies to $\frac{3}{5}$

Add 3 or More Fractions

Notes and Guidance

Children use their knowledge of adding fractions that are multiples of one another to add more than 2 fractions.

They will use an area model and bar models to continue to explore how to add fractions where the denominators are multiples of one another.

Mathematical Talk

How can we split our model?

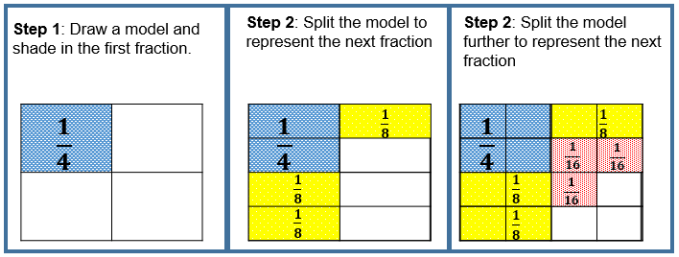
What do you notice about the denominators?

What is that same and what is different about the area model and the bar model?

How do the models show the common denominator?

Varied Fluency

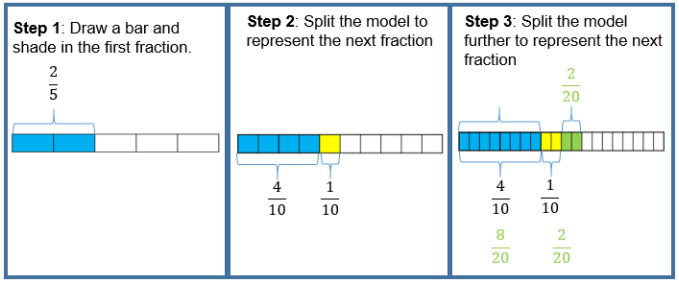
1



Use the area model to solve :

$\frac{1}{3}+\frac{1}{6}+\frac{1}{12}$ $\frac{1}{4}+\frac{3}{8}+\frac{5}{16}$

2



Use the bar model to solve:

$\frac{3}{28}+\frac{3}{14}+\frac{1}{7}$ $\frac{2}{3}+\frac{1}{6}+\frac{1}{12}$

3 Use your preferred method to solve:

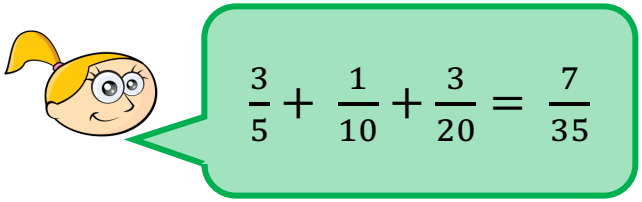
$\frac{3}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{4}$ $\frac{1}{6}+\frac{5}{24}+\frac{3}{12}$ $\frac{7}{36}+\frac{5}{18}+\frac{2}{9}$

Add 3 or More Fractions

Reasoning and Problem Solving

Zoe is attempting to answer:

$$\frac{3}{5}+\frac{1}{10}+\frac{3}{20}$$



Do you agree with Zoe?
Explain why.

Possible answer:
Zoe is wrong because she has added the numerators and denominators together and hasn't found a common denominator.

Sam has added 3 fractions together to get an answer of $\frac{17}{18}$

What 3 fractions could he have added?

Can you find more than one answer?

Children use their knowledge of multiples to find different possible answers.
Some may include:

$$\frac{1}{9}+\frac{5}{9}+\frac{5}{18}$$

$$\frac{1}{6}+\frac{5}{9}+\frac{2}{9}$$

$$\frac{1}{18}+\frac{1}{6}+\frac{13}{18}$$

$$\frac{1}{3}+\frac{1}{6}+\frac{4}{9}$$

Etc.

Add Fractions

Notes and Guidance

Children continue to represent adding fractions using the area model and the bar model to explore adding two or more fractions that are greater than 1

Children can record their totals as an improper fraction but will then convert this to a mixed number using their prior knowledge.

Mathematical Talk

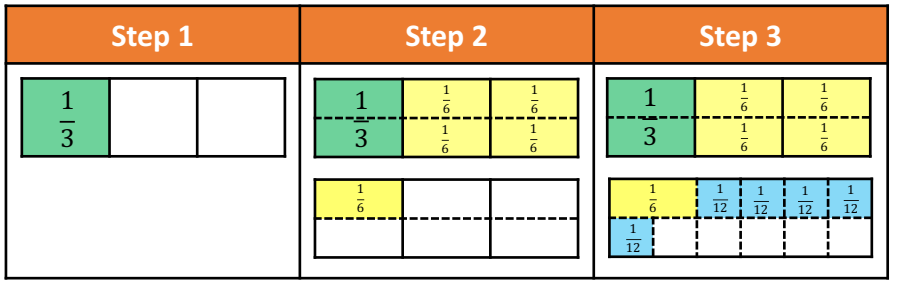
Do I need to count all the sections to find the total? Can you see a whole?

Can we see common equivalent fractions that we already know without converting them?

What is the best way to solve ____ ? Explain why.

Varied Fluency

1

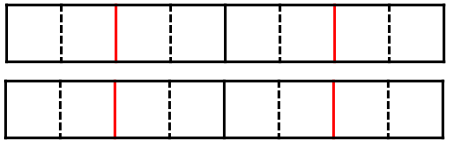


$\frac{1}{3} + \frac{5}{6} + \frac{5}{12} = 1\frac{7}{12}$

Use the area model to help you add the fractions. Give your answer as a mixed number.

$\frac{1}{4} + \frac{2}{3} + \frac{5}{12}$ $\frac{1}{4} + \frac{7}{8} + \frac{3}{16}$

2 Use the bar model to add the fractions. Record your answer as a mixed number.



$\frac{3}{4} + \frac{3}{8} + \frac{1}{2} =$

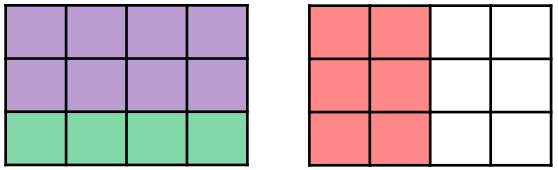
Draw your own models to solve:

$\frac{5}{12} + \frac{1}{6} + \frac{1}{2}$ $\frac{11}{20} + \frac{3}{5} + \frac{1}{10}$ $\frac{3}{4} + \frac{5}{12} + \frac{1}{2}$

Add Fractions

Reasoning and Problem Solving

Gemma is adding three fractions. She uses the model to help her.



What could her three fractions be?

Can you record a number story to represent your calculation?

Possible answer:

$\frac{2}{3} + \frac{4}{12} + \frac{1}{2} = 1\frac{1}{2}$

Other equivalent fractions may be used.

Example story:
Some children are eating pizzas. Jez eats two thirds, Albi eats four twelfths and Dwain it's half a pizza. How much pizza did they eat altogether?

The sum of three fractions is $2\frac{1}{8}$

The fractions have different denominators.

All of the fractions are greater than or equal to a half.

None of the fractions are improper fractions.

All of the denominators are factors of 8

What could the fractions be?

$\frac{1}{2} + \frac{3}{4} + \frac{7}{8}$

Children could be given less clues and explore other possible solutions.

Add Mixed Numbers

Notes and Guidance

Children move on to adding two fractions where one or both are mixed numbers or an improper fraction.

They will use a method of adding the wholes and then adding the parts. Children will record their answer in its simplest form.

Children can still draw models to represent adding fractions.

Mathematical Talk

How can we partition this mixed number into whole numbers and fractions?

What will the wholes total? Can I add the fractions straight away?

What will these mixed numbers be as improper fractions?

If I have an improper fraction in the question, should I change it to a mixed number first? Why?

Varied Fluency

1 $1\frac{1}{3} + 2\frac{1}{6} = 3 + \frac{3}{6} = 3\frac{3}{6}$ or $3\frac{1}{2}$

$1 + 2 = 3$

$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$

Add the fractions by adding the whole first and then the fractions. Give your answer in its simplest form.

$3\frac{1}{4} + 2\frac{3}{8}$ $4\frac{1}{9} + 3\frac{2}{3}$ $2\frac{5}{6} + 2\frac{1}{3}$

2 $1\frac{3}{4} + 2\frac{1}{8} = \frac{7}{4} + \frac{17}{8} = \frac{14}{8} + \frac{17}{8} = \frac{31}{8} = 3\frac{7}{8}$

Add the fractions by converting them to improper fractions.

$1\frac{1}{4} + 2\frac{5}{6}$ $2\frac{1}{9} + 1\frac{1}{3}$ $2\frac{1}{6} + 2\frac{2}{3}$

3 Use your preferred method to add the fractions.

$4\frac{7}{9} + 2\frac{1}{3}$ $\frac{19}{6} + 1\frac{1}{3}$ $\frac{17}{3} + 2\frac{1}{6}$

Add Mixed Numbers

Reasoning and Problem Solving

Joshua and Miriam have some juice.

Joshua drinks $2\frac{1}{4}$ litres and Miriam drinks $2\frac{5}{12}$ litres.

How much do they drink altogether?

Which method would you use and why?

$4\frac{2}{3}$

Encourage children to justify which method they prefer and why. Ensure children discuss which method is more or less efficient.

Fill in the missing numbers.

$$4\ \frac{5}{6} + \square\frac{\square}{\square} = 10\ \frac{1}{3}$$

$5\frac{3}{6}$ *or* $5\frac{1}{2}$

Subtract Fractions

Notes and Guidance

Children subtract fractions with different denominators for the first time, where one denominator is a multiple of the other.

It is important that children see this represented visually so they can make connections with the abstract.

It is important that subtraction is explored as take away and finding the difference.

Mathematical Talk

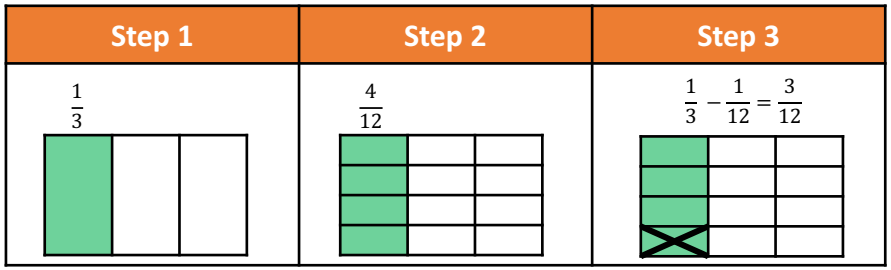
What could the common denominator be?

Can you draw an area model to help you solve the problem?

Is it easier to us a take away bar model or a bar model to find the difference?

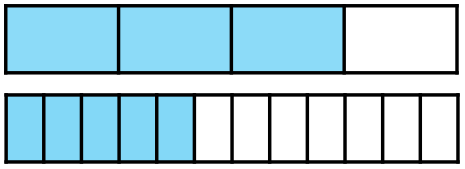
Varied Fluency

1



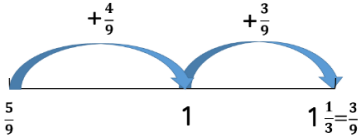
Use an area model to help you solve $\frac{5}{6} - \frac{1}{3}$ and $\frac{7}{8} - \frac{5}{16}$

2



Tom and Hamish both have the same sized chocolate bar. Tom has $\frac{4}{3}$ of the chocolate bar left, Hamish has $\frac{5}{12}$ of the chocolate bar left. How much more does Tom have?

3



Josh uses a number line to find the difference between $\frac{5}{9}$ and $1\frac{1}{3}$

Use this method to find the difference between:

$\frac{3}{4}$ and $\frac{5}{12}$ $1\frac{4}{15}$ and $\frac{3}{5}$ $2\frac{2}{9}$ and $1\frac{1}{3}$

Subtract Fractions

Reasoning and Problem Solving

Which is the odd one out?

A $3\frac{1}{4} - \frac{3}{8}$

B $3\frac{1}{3} - \frac{2}{9}$

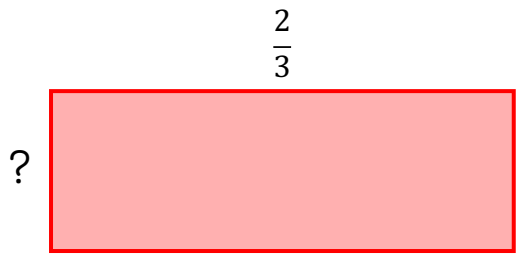
C $3\frac{2}{7} - \frac{1}{3}$

Explain why.

Possible answer:

C is the odd one out because the denominators aren't multiples of each other.

The perimeter of the rectangle is $1\frac{7}{9}$



Work out the missing length.

The missing length is $\frac{2}{9}$

Subtract Mixed Numbers (1)

Notes and Guidance

Children apply their understanding of subtracting fractions where one denominator is a multiple of the other to subtract proper fractions from mixed numbers.

They continue to use models and number lines to support their understanding.

Mathematical Talk

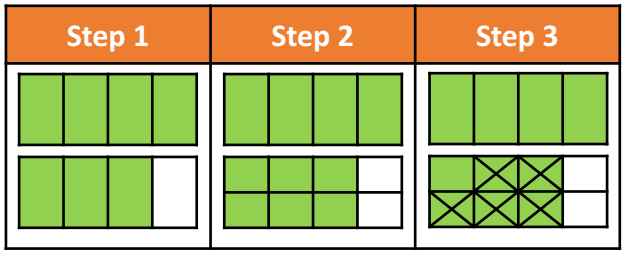
Which fraction is greatest? How do you know?

If the denominators are different, what can we do?

Can you simplify your answer?

Varied Fluency

1



$1\frac{3}{4} - \frac{5}{8} = 1\frac{1}{8}$

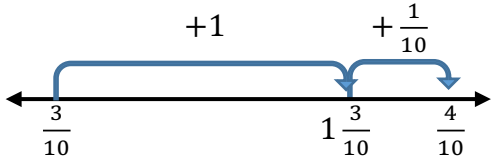
Use an area model to help you solve:

$2\frac{3}{5} - \frac{3}{10}$ $1\frac{2}{3} - \frac{1}{6}$ $1\frac{5}{6} - \frac{7}{12}$

2

$1\frac{2}{5} - \frac{3}{10} = 1\frac{1}{10}$

$1\frac{2}{5} = 1\frac{4}{10}$



Use a number line to solve:

$3\frac{5}{6} - \frac{1}{12}$ $5\frac{5}{7} - \frac{3}{14}$ $2\frac{7}{9} - \frac{11}{18}$

3

Solve:

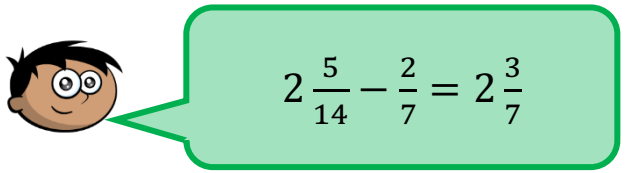
$1\frac{2}{3} - \frac{5}{6}$ $1\frac{3}{4} - \frac{5}{8}$ $2\frac{3}{8} - \frac{11}{16}$

Subtract Mixed Numbers (1)

Reasoning and Problem Solving

Tom is attempting to solve $2\frac{5}{14} - \frac{2}{7}$

Here is his working out:

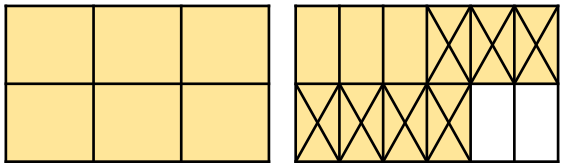


Do you agree with Tom?
Explain your answer.

Possible answer:

Tom is wrong because he hasn't found a common denominator when subtracting the fractions he has just subtracted the numerators and the denominators.

Here is Martha's area model.
What is the calculation?



Can you find more than one answer?
Why is there more than one answer?

The calculation could be $1\frac{5}{6} - \frac{7}{12}$ or $1\frac{10}{12} - \frac{7}{12}$

There is more than one answer because five sixths and ten twelfths are equivalent. Children should be encouraged to write the question as $1\frac{5}{6} - \frac{7}{12}$ so that all fractions are in their simplest form.

Subtract Mixed Numbers (2)

Notes and Guidance

Children use their knowledge of fractions to subtract two fractions where one is a mixed number and you need to break one of the wholes up.

They use the method of flexible partitioning to create a new mixed number so they can complete the calculation.

Mathematical Talk

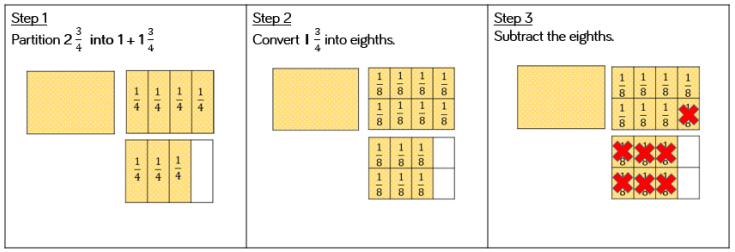
Is flexible partitioning easier than converting the mixed number to an improper fraction?

Do we always have to partition the mixed number?

When can we subtract a fraction without partitioning the mixed number in a different way?

Varied Fluency

1 We can work out $2\frac{3}{4} - \frac{7}{8}$ using this method.



Use this method to calculate:

$3\frac{1}{3} - \frac{5}{6}$ $4\frac{1}{5} - \frac{7}{10}$ $5\frac{2}{3} - \frac{4}{9}$

2 Use flexible partitioning to solve $7\frac{1}{3} - \frac{5}{6}$

$$7\frac{1}{3} - \frac{5}{6} = 6 + 1\frac{1}{3} - \frac{5}{6} = 6 + 1\frac{2}{6} - \frac{5}{6} = 6\frac{3}{6} = 6\frac{1}{2}$$

Use this method to calculate:

$4\frac{2}{3} - \frac{5}{6}$ $4\frac{1}{5} - \frac{7}{15}$ $5\frac{1}{4} - \frac{7}{8}$

3 Mr Brown has $3\frac{1}{4}$ bags of flour. He uses $\frac{6}{8}$ of a bag. How much flour does he have left?

Subtract Mixed Numbers (2)

Reasoning and Problem Solving

Place 2, 3 and 4 in the boxes to make the calculation correct.

$$27\frac{1}{\square} - \frac{\square}{6} = 26\frac{\square}{3}$$

$27\frac{1}{3} - \frac{4}{6} = 26\frac{2}{3}$

3 children are working out $6\frac{2}{3} - \frac{5}{6}$

They partition the mixed number in the following ways to help them.

Lucy $5 + 1\frac{2}{3} - \frac{5}{6}$

Mary $5 + 1\frac{4}{6} - \frac{5}{6}$

Sam $5 + \frac{10}{6} - \frac{5}{6}$

Whose method is correct?
Explain why.

All three children are correct.

$1\frac{2}{3}$, $1\frac{4}{6}$ and $\frac{10}{6}$ are all equivalent therefore all three methods will help children to correctly calculate the answer.

Subtract 2 Mixed Numbers

Notes and Guidance

Children use different strategies to subtract two mixed numbers.

Building on learning in previous steps, they look at partitioning the mixed numbers into wholes and parts and build on their understanding of flexible partitioning to subtract two mixed numbers when an exchange is involved.

Mathematical Talk

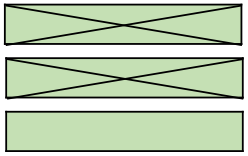
Why is subtracting the wholes and parts separately easier with some fractions than others?

Can you show the subtraction as a difference as well as a take away on the bar model?

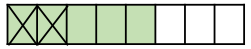
Does making the whole numbers larger make the subtraction any more difficult? Explain why.

Varied Fluency

1 Here is a bar model to calculate $3\frac{5}{8} - 2\frac{1}{4}$



$3 - 2 = 1$



$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

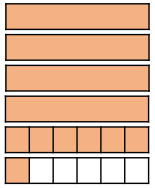
$3\frac{5}{8} - 2\frac{1}{4} = 1\frac{3}{8}$

Use this method to calculate:

$3\frac{7}{8} - 2\frac{3}{4}$ $5\frac{5}{6} - 2\frac{1}{3}$ $3\frac{8}{9} - 2\frac{5}{27}$

Why does this method not work effectively for $5\frac{1}{6} - 2\frac{1}{3}$?

2 Here is a method to calculate $5\frac{1}{6} - 2\frac{1}{3}$



$5\frac{1}{6} - 2\frac{1}{3} = 4\frac{7}{6} - 2\frac{1}{3} = 4\frac{7}{6} - 2\frac{2}{6} = 2\frac{5}{6}$

Use this method to calculate:

$3\frac{1}{4} - 2\frac{5}{8}$ $5\frac{1}{3} - 2\frac{7}{12}$ $27\frac{1}{3} - 14\frac{7}{15}$

Subtract 2 Mixed Numbers

Reasoning and Problem Solving

There are three types of chocolate in a chocolate box: milk chocolate, dark chocolate and white chocolate.

The total mass of the chocolate is 7 kg.

The mass of milk chocolate is $3\frac{3}{4}$ kg and the mass of dark chocolate is $1\frac{7}{16}$ kg.

What is the mass of white chocolate?

$3\frac{3}{4} + 1\frac{7}{16} = 5\frac{3}{16}$

$7 - 5\frac{3}{16} = 1\frac{13}{16}$

The mass of white chocolate is $1\frac{13}{16}$ kg.

Rachel has $135\frac{2}{5}$ cm of ribbon.

Kyra has $2\frac{11}{15}$ cm less ribbon than Rachel.

How much ribbon do they have altogether?

Kyra has $132\frac{10}{15}$ cm of ribbon.

Children should simplify this to $132\frac{2}{3}$ cm.

Altogether they have $268\frac{1}{15}$ cm of ribbon.

Multiply by an Integer (1)

Notes and Guidance

Children are introduced to multiplying fractions by a whole number for the first time. They link this to repeated addition and see that the denominator remains the same, whilst the numerator is multiplied by the integer.

This is shown clearly through the range of models to build the children's conceptual understanding of multiplying fractions.

Mathematical Talk

How is multiplying fractions similar to adding fractions?

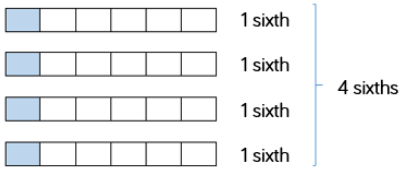
Which bar model do you find the most useful?

Which bar model helps us to convert from an improper fraction to a mixed number most effectively?

Varied Fluency

1 Work out $\frac{1}{6} \times 4$ by counting in sixths.

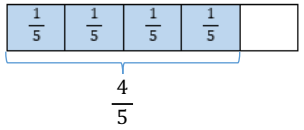
$\frac{1}{6} \times 4 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$



Use this method to work out:

$\frac{1}{3} \times 2$ $\frac{1}{5} \times 3$ $\frac{1}{10} \times 6$

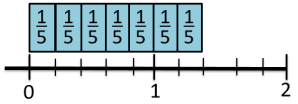
2 We can use a single bar model to work out $\frac{1}{5} \times 4$



Use this method to work out:

$\frac{1}{4} \times 3$ $\frac{1}{8} \times 6$ $\frac{1}{10} \times 8$

3 We can use a number line and repeated addition to work out $\frac{1}{5} \times 7 = \frac{7}{5} = 1\frac{2}{5}$



Use this method to work out:

$\frac{1}{3} \times 3$ $\frac{1}{7} \times 6$ $\frac{1}{12} \times 8$

Multiply by an Integer (1)

Reasoning and Problem Solving

Ranjit is multiplying fractions by a whole number.

$\frac{1}{5} \times 5 = \frac{5}{25}$

Can you explain his mistake?

Possible answer:
Ranjit has multiplied the numerator and the denominator rather than recognising that he has five lots of one fifth.
He has found an equivalent fraction.

Always, sometimes, never.

When you multiply a unit fraction by the same number as it's denominator the answer will be one whole.

Always because your numerator will be the same as your denominator which means that it is a whole.
E.g. $\frac{1}{3} \times 3 = \frac{3}{3} = 1$

I am thinking of a unit fraction.

When I multiply it by 4 it will be equivalent to $\frac{1}{2}$

When I multiply it by 2 it will be equivalent to $\frac{1}{4}$

What is my fraction?

What do I need to multiply it by so that my answer is equivalent to $\frac{3}{4}$

Can you create your own version of this problem?

$\frac{1}{8}$ because
$4 \times \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$
and
$2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

6 because
$6 \times \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$

Multiply by an Integer (2)

Notes and Guidance

Children apply prior knowledge of multiply a fraction by a whole number to multiplying a non-unit fraction by a whole number.

They use similar models and discuss which method will be the most efficient depending on the questions asked.

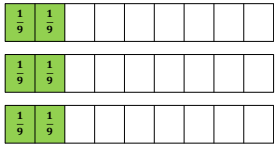
Mathematical Talk

Can you show me 3 lots of $\frac{3}{10}$ on a bar model?
How many tenths do we have altogether?

How does repeated addition help us with this multiplication?
How does a number line help us see the multiplication?

Varied Fluency

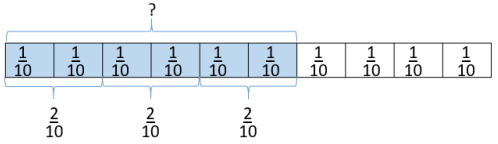
1 Count the number of ninths to work $3 \times \frac{2}{9}$



Use this method to work out:

$\frac{3}{8} \times 2$ $\frac{5}{16} \times 3$ $4 \times \frac{2}{11}$

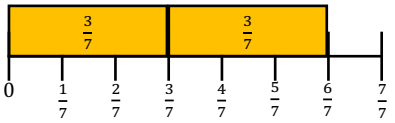
2 Use the model to help you solve $3 \times \frac{2}{10}$



Use this method to work out:

$\frac{2}{7} \times 3$ $\frac{3}{16} \times 4$ $2 \times \frac{5}{12}$

3 Use the number line to help you solve $2 \times \frac{3}{7}$



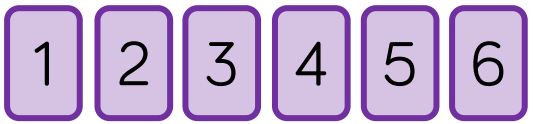
Use this method to work out:

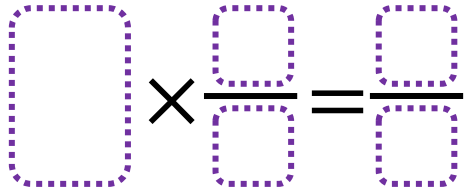
$\frac{3}{10} \times 3$ $\frac{2}{7} \times 2$ $4 \times \frac{3}{20}$

Multiply by an Integer (2)

Reasoning and Problem Solving

Use the digit cards to complete the multiplication.





Possible answer:
Ranjit has multiplied the numerator and the denominator rather than recognising that he has five lots of one fifth.
He has found an equivalent fraction.

Always because your numerator will be the same as your denominator which means that it is a whole.
E.g. $\frac{1}{3} \times 3 = \frac{3}{3} = 1$

Denise has calculated $4 \times \frac{3}{14}$

From the picture I can see that $4 \times \frac{3}{14} = \frac{12}{56}$

Do you agree?

Explain why.

Possible answer:

I disagree. Denise has shaded 12 fourteenths. She has counted all of the boxes to give her the denominator when she shouldn't have. The answer should be $\frac{12}{14}$ or $\frac{6}{7}$

Multiply by an Integer (3)

Notes and Guidance

Children use their knowledge of fractions to multiply a mixed number by a whole number.

They use the method of multiplying the whole and part separately and also the method of converting to an improper fraction then multiplying.

It is important that they see varied representations of fractions.

Mathematical Talk

How could you represent this mixed number?
What is the denominator? How do you know?
How many wholes are there? How many parts are there?
What is multiplying fractions similar to? (repeated addition)
What representation could you use to convert a mixed number to an improper fraction?

Varied Fluency

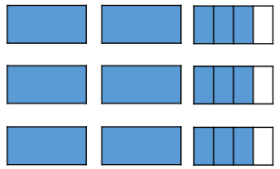
1 Use repeated addition to work out $2\frac{2}{3} \times 4$

$$2\frac{2}{3} \times 4 = 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} = 8\frac{8}{3} = 10\frac{2}{3}$$

Use this method to solve

$2\frac{1}{6} \times 3$ $\quad 1\frac{3}{7} \times 2$ $\quad 3\frac{1}{3} \times 4$

2 Partition your fraction to help you solve $2\frac{3}{4} \times 3$



- $2 \times 3 = 6$
- $\frac{3}{4} \times 3 = \frac{9}{4} = 1\frac{3}{4}$
- $6 + 1\frac{3}{4} = 7\frac{3}{4}$

Use this method to answer

$2\frac{5}{6} \times 3$ $\quad 3\frac{4}{7} \times 2$ $\quad 2\frac{1}{3} \times 5$

3

$$1\frac{5}{6} \times 3 = \frac{11}{6} \times 3 = \frac{33}{6} = 5\frac{3}{6} = 5\frac{1}{2}$$

Convert to an improper fraction to work out

$3\frac{2}{7} \times 4$ $\quad 2\frac{4}{9} \times 2$ $\quad 4 \times 3\frac{3}{5}$

Multiply by an Integer (3)

Reasoning and Problem Solving

Jack runs $2\frac{2}{3}$ miles three times per week.

Josh runs $3\frac{3}{4}$ miles twice a week.

Who runs the furthest during the week?

Explain your answer.

Jack runs
$2\frac{2}{3} \times 3 = 8$ miles

Josh runs
$3\frac{3}{4} \times 2 = 7\frac{1}{2}$ miles

Jack runs further by half a mile.

Work out the missing numbers.

$$2\frac{\square}{8} \times \square = 7\frac{7}{8}$$

Explain how you worked it out.

Possible answer:

$2\frac{5}{8} \times 3 = 7\frac{7}{8}$

I knew that the multiplier could not be 4 because that would give an answer of at least 8. So the multiplier had to be 3. That meant that the missing numerator had to give a product of 15. I knew that 5 multiplied by 3 would give 15

Fraction of an Amount

Notes and Guidance

Children find unit and non-unit fractions of amount, quantities and measures.

It is important that the concept is explored pictorially through bar models to support children to make sense of the abstract.

Mathematical Talk

How many equal groups have you shared 49 into? Why?

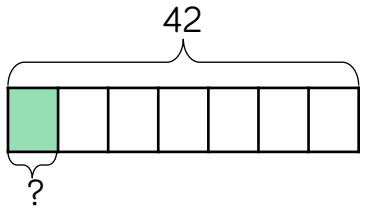
What does each equal part represent as a fraction and an amount?

What could you do to 1 metre to make the calculation easier?

Could you convert $\frac{4}{5}$ to make the calculation any easier?

Varied Fluency

1 Find $\frac{1}{7}$ of 42



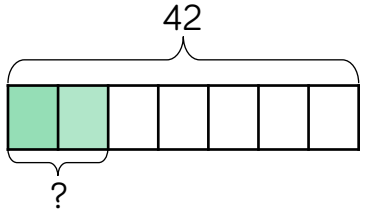
$42 \div 7 = 6$

$\frac{1}{7}$ of 42 is 6

Use this method to find

$\frac{1}{8}$ of 56 $\frac{1}{6}$ of 480 $\frac{1}{9}$ of 81 m

2 Find $\frac{2}{7}$ of 42



$42 \div 7 = 6$

$6 \times 2 = 12$

$\frac{2}{7}$ of 42 is 12

Use this method to find

$\frac{3}{8}$ of 56 $\frac{5}{6}$ of 480 $\frac{4}{9}$ of 81 m

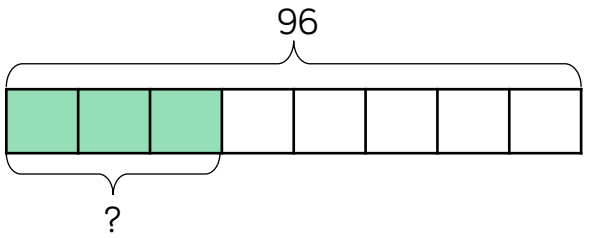
3 Draw a bar model to help you calculate

$\frac{4}{5}$ of 1 m $\frac{5}{12}$ of 1.44 litres $\frac{3}{7}$ of 21 kg

Fraction of an Amount

Reasoning and Problem Solving

Write a problem that matches the bar model.



Possible response:

There are 96 cars in a car park.
$\frac{3}{8}$ of them are red.
How many cars are red?

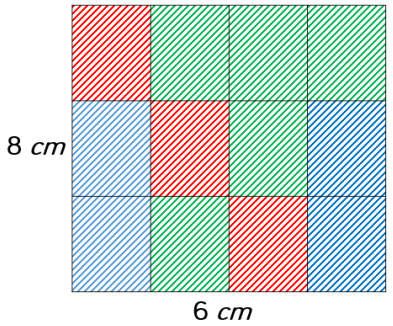
$\frac{7}{16}$ of a class are boys.

There are 18 girls in the class.

How many children are in the class?

There are 32 children in the class.

Find the area of each colour in the rectangle.



Area of rectangle:
$6 \times 8 = 48$ cm^2

Blue
$\frac{4}{12}$ of $48 = 16$ cm^2

Red
$\frac{3}{12}$ of $48 = 12$ cm^2

Green
$\frac{5}{12}$ of $48 = 20$ cm^2

Fractions as Operators

Notes and Guidance

Children link their understanding of fractions of amounts and multiplying fractions to use fractions as operators.

They use their knowledge of commutativity to help them understand that you can change the order of multiplication without changing the outcome.

Mathematical Talk

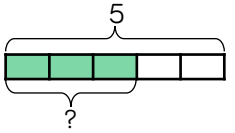
Is it easier to multiply a fraction of find a fraction of an amount? Does it depend on the whole number you are multiplying by?

Can you see the link between the numbers? Can you use previous calculations to help you calculate missing numbers?

Varied Fluency

1 Jenny has calculated and drawn a bar model for two calculations.

$5 \times \frac{3}{5} = \frac{15}{5} = 3$ 

$\frac{3}{5}$ of $5 = 3$ 

What's the same and what's different about Jenny's calculations?

2 Complete:

2 lots of $\frac{1}{10} = \square$ $\frac{1}{10}$ of $2 = \square$

8 lots of $\square = 200$ 8 lots of $2.5 = \square$

8 lots of $\frac{1}{4} = \square$ $\frac{1}{4}$ of $8 = \square$

3 Use this to complete:

$20 \times \frac{4}{5} = \square$ of $20 = \square$ $\square \times \frac{2}{3} = \square$ of $18 = 12$

$\square \times \frac{1}{3} = \frac{1}{3}$ of $\square = 20$

Fractions as Operators

Reasoning and Problem Solving

Which calculations are easier to multiply the fractions, and which are easier to find the fraction of an amount?
Explain you choice for each one.

$25 \times \frac{3}{5}$ or $\frac{3}{5}$ of 25

$6 \times \frac{2}{3}$ or $\frac{2}{3}$ of 6

$5 \times \frac{3}{8}$ or $\frac{3}{8}$ of 5

Possible response:

1. Children may find it easier to find 3 fifths of 5 rather than multiply 25 by 3
2. Children may choose either as they are of similar difficulty.
3. Children will probably find it easier to multiply than divide 5 by 8

Jamie and Sam are thinking of a two-digit number between 20 and 30

Jamie finds two thirds of the number

Sam multiplies the number by $\frac{2}{3}$

Their new two-digit number has a digit total that is one more than that of their original number

What number did they start with?

Show each step of their calculation.

They started with 24

Jamie:
$24 \div 3 = 8$
$8 \times 2 = 16$

Sam:
$24 \times 2 = 48$
$48 \div 3 = 16$